Problem 3

Evaluate $\lim_{x\to 0} \frac{|2x-1|-|2x+1|}{x}$.

Solution

Evaluate the limit.

$$\begin{aligned}
\lim_{x\to 0} \frac{|2x-1|-|2x+1|}{x} &= \lim_{x\to 0} \frac{\sqrt{(2x-1)^2}-\sqrt{(2x+1)^2}}{x} \\
&= \lim_{x\to 0} \frac{\sqrt{(2x-1)^2}-\sqrt{(2x+1)^2}}{x} \times \frac{\sqrt{(2x-1)^2}+\sqrt{(2x+1)^2}}{\sqrt{(2x-1)^2}+\sqrt{(2x+1)^2}} \\
&= \lim_{x\to 0} \frac{\left[\sqrt{(2x-1)^2}-\sqrt{(2x+1)^2}\right]\left[\sqrt{(2x-1)^2}+\sqrt{(2x+1)^2}\right]}{x\left[\sqrt{(2x-1)^2}+\sqrt{(2x+1)^2}\right]} \\
&= \lim_{x\to 0} \frac{(2x-1)^2-(2x+1)^2}{x\left[\sqrt{(2x-1)^2}+\sqrt{(2x+1)^2}\right]} \\
&= \lim_{x\to 0} \frac{(4x^2-4x+1)-(4x^2+4x+1)}{x\left[\sqrt{(2x-1)^2}+\sqrt{(2x+1)^2}\right]} \\
&= \lim_{x\to 0} \frac{-8x}{x\left[\sqrt{(2x-1)^2}+\sqrt{(2x+1)^2}\right]} \\
&= \lim_{x\to 0} \frac{-8}{\sqrt{(2x-1)^2}+\sqrt{(2x+1)^2}} \\
&= \frac{-8}{\sqrt{(-1)^2}+\sqrt{(1)^2}} \\
&= -4
\end{aligned}$$